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A STUDY ON MARKOWITZ QUADRATIC PROGRAMMING MODEL AND COMPARATIVE STUDY WITH MARKET INDEX PORTFOLIOS

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ABSTRACT

This study constructs portfolios from the stocks of Bombay Stock Exchange using Markowitz Quadratic Programming model and then compares it with the market index portfolios. It analyzes the portfolio performance with varying holding periods and also deals with the problem of determining the optimal holding period for Markowitz portfolios as well as market portfolios and compares them. The paper also analyses portfolios sector-wise and explains the results of the optimal holding periods for the indices.

KEYWORDS: Stock Exchange, Markowitz Quadratic, Portfolios

INTRODUCTION

According to the portfolio theory (Markowitz, 1952), investors require a higher return from the market portfolio than from risk free return investments. This market portfolio return depends on risk indicating a positive relationship. Merton (1973) shows that the conditional expected excess return on the aggregate stock market is a linear function of its conditional variance with a positive slope. French et al. (1987), Campbell (1987), Chou (1988), Chan et al. (1992), Chou, et al. (1992), Glosten et al. (1993), Harvey (1989, 2001), Bollerslev and Zhou (2005) and Ludvigson and Ng (2007) used daily data in order to examine the risk - return relationship with most of these studies to support the expected positive relationship.

PREVIOUS RESEARCH

Ulucan (2007) investigated the optimal holding period for the Mean Variance efficient portfolio using Istanbul stocks. ISE-100$^{\#}$ index and FTSE-40 index stocks data between January 2000 and November 2004 were examined. They adopted Mean Variance(MV), Semi Variance(SV) and Expected Loss(EL) as risk criteria to solve the optimization problems. The empirical results indicated as follows: MV efficient investment portfolio performs better in longer term investment horizons. The 9-month holding period provides the best performance but this advantage will no longer exist once the holding period is greater than 12-15 months.

Mu-Lan Wang et al. (2010) analyzed the portfolio performance with varying holding periods using Taiwan stocks. He used the Taiwan 50 Index, Taiwan Mid-Cap 100 Index, Taiwan Technology Index and Finance Index stocks data from January 2005 to March 2009 as samples. The performance for the MV method is better than the index return, which shows that more active investment strategy provides a better return compared with the index. Additionally, the optimal holding period is 2-6 months.

Ali Argun Karacabey (2006) compares Markowitz model with mean variance as risk measure and three other portfolio optimization models which use mean absolute deviation as risk measure. In order to evaluate the performance of the portfolio optimization models, he used 5 year data –from January 2000 to December 2004- which contain monthly adjusted price information for securities involved in ISE-100, the well known index of Istanbul Stock Exchange. Mean variance portfolios under the 3 month assumption produced higher returns than the market and the mean absolute deviation portfolios. In the case of portfolios being revised in every 6 months, MAD portfolios showed bigger progress than MV portfolios but both of them still underperformed the market. Shifting the portfolio composition one in a year does not make any statistically significant difference for the MAD portfolios but it decreased the performance of the MV portfolios.

Parada (2008) develops some propositions for building a portfolio made up of risky assets to substitute a risk-free asset, further determining the proportions that should be invested to generate this portfolio and analyzing the construction of a portfolio to substitute the market portfolio.

Konno (1991) compared the performance of his optimization model with mean absolute deviation as risk measure with that of Markowitz model using the historical data of 224 stocks included in NIKKEI 225 index and TOPIX index. Markowitz portfolios always outperformed the market portfolios and appeared to be somewhat better than portfolios constructed from his model. Portfolios with MAD as risk measure were better when compared to market portfolios for most of the time. Portfolio models were comparable to Markowitz model when the number of stocks is on the higher side so can be used practically. Calculated optimal portfolios and their performance were quite similar to Markowitz portfolios and believe that these portfolios will not be very much different for the model when the number of stocks exceeds 1000.

Kroll et al. (1984) reported that the mean-variance portfolio has a maximum utility function or at least a near optimum expected utility. The solution of an optimization problem is the vector of portfolio weights, i.e. parts of the investor's wealth invested into the selected assets.

RESEARCH METHOD AND DATA SOURCE

The basic portfolio model developed by Harry Markowitz derived the expected rate of return for a portfolio of assets and an expected risk measure. Markowitz showed that the variance of the rate of return was a meaningful measure of portfolio risk under a reasonable set of assumptions, and derived the formula for computing the variance of a portfolio. The Markowitz model is based on several assumptions* regarding investor behavior under which, a single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk, or lower risk with the same (or higher) expected return.

Markowitz portfolio optimization model employs variance as the measure of risk and the objective of the model is to find out the weightings of the assets that minimize the variance of a portfolio and provide the portfolio to have a return equal or bigger than the expected return. So the mathematical model for *n* assets is as follows:

The expected return for the portfolio

$$E(R_{port}) = \sum_{i=1}^{n} W_i E(R_i)$$

For two assets, *i* and *j*, the covariance of rates of return is defined as:

$$Cov_{ij} = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

Standard Deviation of a portfolio as follows[1]

$$\sqrt{\left(\sum_{i=1}^{n} W_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} W_i W_j Cov_{ij}\right)}$$

*The assumptions are given below:

- Investors consider each investment alternative as being represented by a probability distribution of expected returns over some holding period.
- Investors maximize one-period expected utility, and their utility curves demonstrate diminishing marginal utility of wealth.
- Investors estimate the risk of the portfolio on the basis of the variability of expected returns.
- Investors base decisions solely on expected return and risk, so their utility curves are a function of expected return and the expected variance (or standard deviation) of returns only.
- For a given risk level, investors prefer higher returns to lower returns. Similarly, for a given level of expected return, investors prefer less risk to more risk

Minimize

$$\sum_{i=1}^{n}\sum_{j=1}^{n} Cov_{ij} W_i W_j$$

Subject to

$$\sum_{i=1}^{n} R_i W_i \geq \rho$$

$$\sum_{i=1}^{n} W_i = 1$$

$$0 \leq W_i \leq 1 \qquad i = 1 \ldots . n$$

Where,

W_i = the percent of the portfolio in asset i

$E(R_i)$ = the expected rate of return for asset i σ_{port} = the standard deviation of the portfolio σ_i^2 = the variance of rates of return for assets i Cov_{ij} = the covariance between the rates of return for assets i and j

ρ = a parameter representing the minimal rate of return required by an investor

Fabozzi (1999), in order to construct an efficient portfolio in the Markowitz model could be summarized as follows, one needs to

- Calculate the expected return rates for each stock to be included in the portfolio,
- Calculate the variance or standard deviation (risk) for each stock to be included in the portfolio,
- Calculate the covariance or correlation coefficients for all stocks, treating them as pairs.

The model yields the optimum weights or the percentage of investment in each asset in a portfolio which allows the investor to have maximum returns at a minimum risk. By varying the weights the investor can notice the changes in the returns and risks associated with the portfolios and accordingly alter his investments in line with how risk averse he/she is.

The present study analyzes the stock data of 7 indices of the Bombay Stock Exchange (BSE)[1] out of which two of them are broad indices, BSE-SENSEX and BSE-MIDCAP. The study targeted five sector wise indices AUTO, FMCG, IT, METAL and OIL GAS. The selected study period is between the dates January 2004 to December 2006, during which the average of opening and closing values of the stocks of the relevant firms and the indices were employed as the study data. The data obtained average return values for 3 years for each stock and index on a monthly basis when the performance was evaluated for a span of 6 months or more and on a daily basis when the portfolios were constructed for a span of 3 months or less.

In the case of evaluation of performance of portfolios of the broad Indices SENSEX and MIDCAP, the firms were chosen through randomly in order to ensure objectivity. Ten stocks from each index were chosen and then compared with the respective index.

In the case of evaluation of performance of portfolios of the sector wise indices, AUTO comprises of 11 automobile industries but due to lack of data 10 stocks were only considered in the portfolio. FMCG consists of 10 fast moving consumer goods firms in the portfolio. IT comprises of 10 firms out of which 9 were considered in the portfolio because of missing data of 1 firm. METAL portfolio has 12 stocks out of 13 firms listed in the index. OIL GAS consists of 7 firms in the portfolio out of the 9 listed firms in the index.

The risk-free rate of return has been taken as the rate of interest offered to a fixed deposit in government banks which turns out to be 3percent.

Returns[+] are calculated from the stock price data obtained on a monthly basis for portfolios whose holding periods are atleast 6 months and on a daily basis for portfolios whose holding periods are 3 months or less. The Covariance matrix of a portfolio was obtained using covariance function in excel solver. The drawings of an efficient frontier for a portfolio of stocks was done using Monte Carlo simulation in excel. A tangent drawn from the taken risk-free rate of interest to the frontier gives the efficient portfolio which means we get the optimum weights of each stock in a portfolio

[1]Return is defined as $R_i = \frac{EV-BV}{BV}$; EV is ending value, BV is beginning value

[*] Data collected from www.bseindia.com

EMPIRICAL RESULTS

The performances for Markowitz efficient frontiers of all the 7 indices are compared with corresponding index

returns in 1, 3, 6, 9, 12, 18, 24 and 36-month holding periods. In Tables 1 to 7, the returns of the Markowitz portfolios for all target return levels are superior to the index returns for any holding period.

In Table 1, the returns of the optimal portfolio are clearly higher than that of the index SENSEX returns and returns are a maximum with the lowest risk at a holding period of 3 months. This shows that reviewing the portfolios for every 3 months time period is expected to be much more profitable to the investor. In Table 3 and Table 7, the optimal holding periods for the portfolios made up of stocks from index AUTO and OIL GAS respectively turned out to be 3 months which clearly shows the speculative nature of these stocks and active trading could give higher returns.

From tables 2, 4, 5 and 6, the derived optimal holding period for portfolios consisting of stocks from indices MIDCAP, FMCG, IT and METAL is 12 months. Industries in the METAL index have longer gestation periods and economic life cycles which could account to their 12 month optimum period to give higher returns.

The portfolio of stocks from the BSE-IT index returned the highest yield and their optimum period is 12 months which shows that though these stocks are volatile in nature they remain at certain levels either high or low for longer periods of time when compared to much more speculated stocks. The lifecycle of the projects of the IT firms also account to the longer holding period even though they are volatile in nature.

Table 1: Efficient Portfolios' Performance over BSE-SENSEX Index Returns (Jan '04–Dec '06)

Holding Period	Index		Optimal Portfolio	
In Months	Mean	Standard Deviation	Mean	Standard Deviation
1	-0.14	1.85	0.57	2.44
3	-1.87	1.71	4.06	0.09
6	-1.53	5.21	0.95	7.01
9	-4.96	14.72	1.82	5.92
12	-1.74	13.5	3.32	5.52
18	-0.44	11.11	3.17	5.36
24	1.01	9.88	3.84	5.25
36	1.08	8.45	3.65	5.36

Table 2: Efficient Portfolios' Performance over BSE-MIDCAP Index Returns (Jan '04–Dec '06)

Holding Period	Index		Optimal Portfolio	
In Months	Mean	Standard Deviation	Mean	Standard Deviation
1	-0.71	2.52	-0.69	3.4
3	-8.79	1.91	-4.49	4.91
6	-6.77	2.84	0.07	9.38
9	-0.59	9.9	4.89	10.9
12	1.67	10.06	5.2	8.82
18	1.1	8.36	5.09	9.19
24	1.53	7.92	4.71	7.17
36	2.16	8.5	4.07	8.45

Table 3: Efficient Portfolios' Performance over BSE-AUTO Index Returns (Jan '04–Dec '06)

Holding Period	Index		Optimal Portfolio	
In Months	Mean	Standard Deviation	Mean	Standard Deviation
1	-0.04	2.34	0.29	2.68
3	0.82	0.98	4.19	1.64
6	-1.91	6.38	0.41	7.52
9	-0.86	5.09	1.06	7.47
12	-0.04	6.03	1.87	5.15
18	0.88	5.72	2.18	5.77
24	0.23	6.07	1.95	4.96
36	0.68	5.73	1.91	5.42

Table 4: Efficient Portfolios' Performance over BSE-FMCG Index Returns (Jan '04–Dec '06)

Holding Period	Index		Optimal Portfolio	
In Months	Mean	Standard Deviation	Mean	Standard Deviation
1	-0.19	1.73	-0.1	2.64
3	-1.46	1.34	-0.52	5.46
6	-6.41	8.82	-3.59	3.9
9	-0.53	5.16	1.24	7.03
12	1.96	6.78	7.07	8.05
18	2.73	5.61	6.07	8.91
24	2.06	6.97	5.47	5.92
36	1.39	8	4.67	9.39

Table 5: Efficient Portfolios' Performance over BSE-IT Index Returns (Jan '04–Dec '06)

Holding Period	Index		Optimal Portfolio	
In Months	Mean	Standard Deviation	Mean	Standard Deviation
1	-0.37	1.35	2.34	2.15
3	-4.6	2.63	3.07	1.77
6	-2.21	3.58	6.63	4.21
9	-6.13	9.7	9.7	7.9
12	-3.11	15.13	12.24	17.7
18	-1.1	12.57	11.28	19.2
24	-0.09	11.01	7.15	8.54
36	0.24	9.36	7.03	12.6

Table 6: Efficient Portfolios' Performance over BSE-METAL Index Returns (Jan '04–Dec '06)

Holding Period	Index		Optimal Portfolio	
In Months	Mean	Standard Deviation	Mean	Standard Deviation
1	-0.9	2.94	-0.52	1.58
3	-12.54	0.41	-5.77	3.99
6	-10.29	4.41	-7.78	5.27
9	-2.66	11.15	0.67	10.7
12	0.51	10.96	3.36	11
18	0.22	9.5	2.73	9.2
24	0.81	8.4	3.02	8.59
36	1.81	9.99	3.22	10.7

Table 7: Efficient Portfolios' Performance over BSE-OIL GAS Index Returns (Jan '04–Dec '06)

Holding Period	Index		Optimal Portfolio	
In Months	Mean	Standard Deviation	Mean	Standard Deviation
1	0.12	2.66	0.31	3.32
3	0.04	6.29	5.44	1.05
6	-5.33	9.87	2.61	7.92
9	-1.84	8.81	1.48	5.85
12	0.06	8.07	5.5	8.12
18	0.15	6.73	4.05	7.01
24	0.95	6.12	4.12	6.76
36	0.56	5.78	2.18	5.63

CONCLUSIONS

The quadratic programming model of Markowitz has been tested with the real data of an emerging market and the analyses showed that the Markowitz portfolios always outperformed the index portfolios. In any holding periods Markowitz portfolios gave higher returns compared to the market portfolios. The model reduces the difficulty in assessing the differential risks or variations when it is a case of more number of assets in the portfolio.

The analysis is carried out on different sectors in the economy and the results show where the investors could gain more and where to invest. The IT sector had high returns and a 12 month optimal holding period whereas the OIL GAS sector showed a 3 month optimal period which reflects that an investor could expect more returns through much more active trading than compared to the IT stocks.

These results suggest that the revision of portfolios using Markowitz model at the corresponding optimal holding periods of the respective sectors would profit the investors.

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APPENDICES

The stocks included in the BSE-SENSEX index are:

Company Name	Scrip Id	Scrip Code
Bharti Airtel Ltd.	Bhartiartl	532454
Bharat Heavy Electricals Ltd.	Bhel	500103
Hdfc Bank Ltd.	Hdfc	500180
Infosys Technologies Ltd.	Infosys	500209
Larsen & Toubro Ltd.	Lnt	500510
Oil And Natural Gas Corporation Ltd.	Ongc	500312
Reliance Industries Ltd.	Ril	500325
State Bank Of India	Sbi	500112
Tata Motors Ltd.	Tatamotors	500570
Tata Steel Ltd.	Tatastl	500470

The stocks included in the BSE-MIDCAP index are:

Company Name	Scrip Id	Scrip Code
Andhra Bank	Andhrabank	532418
Birla Corporation Ltd.	Birlacorp	500338
Bombay Dyeing & Mfg.Co.Ltd.	Bombaydy	500020
Essar Shipping Ports & Logistics Ltd.	Essarship	500630
Glaxosmithkline Consumer Healthcare Ltd.	Glaxocon	500676
Hindustan Oil Exploration Co.Ltd.	Hindoilexp	500186
Ing Vysya Bank Ltd.	Ingvysya	531807
Mrf Ltd.	Mrf	500290
Mahanagar Telephone Nigam Ltd.	Mtnl	500108
Zuari Industries Ltd.	Zuariind	500780

The stocks included in the BSE-AUTO index are:

Company Name	Scrip Id	Scrip Code
Amtek Auto Ltd.	Amtek	520077
Apollo Tyres Ltd.	Apollotyre	500877

Ashok Leyland Ltd.	Ashokley	500477
Bharat Forge Ltd.	Bharatforg	500493
Cummins India Ltd.	Cummins	500480
Exide Industries Ltd.	Exideind	500086
Hero Honda Motors Ltd.	Herohonda	500182
Mahindra & Mahindra Ltd.	Mnm	500520
Maruti Suzuki India Ltd.	Maruti	532500
Tata Motors Ltd.	Tatamotors	500570

The stocks included in the BSE-FMCG index are:

Company Name	**Scrip Id**	**Scrip Code**
Colgate-Palmolive (India) Ltd.	Colgate	500830
Dabur India Ltd.	Dabur	500096
Godrej Consumer Products Ltd.	Godrejcp	532424
Hindustan Unilever Ltd.	Hul	500696
Itc Ltd.	Itc	500875
Marico Ltd.	Marico	531642
Nestle India Ltd.	Nestle	500790
Tata Global Beverages Ltd.	Tataglobal	500800
United Breweries Ltd.	Unitedbrew	532478
United Spirits Ltd.	Unitdspr	532432

The stocks included in the BSE-OIL GAS index are:

Company Name	**Scrip Id**	**Scrip Code**
Bharat Petroleum Corporation Ltd.	Bpcl	500547
Gail (India) Ltd.	Gail	532155
Hindustan Petroleum Corporation Ltd.	Hpcl	500104
Indian Oil Corporation Ltd.	Ioc	530965
Oil And Natural Gas Corporation Ltd.	Ongc	500312
Petronet Lng Ltd.	Petronet	532522
Reliance Industries Ltd.	Ril	500325

The stocks included in the BSE-IT index are:

Company Name	Scrip Id	Scrip Code
Core Projects & Technologies Ltd.	Coreproject	512199
Financial Technologies (India) Ltd.	Fintech	526881
Hcl Technologies Ltd.	Hcltech	532281
Infosys Technologies Ltd.	Infosys	500209
Mphasis Ltd.	Mphasis	526299
Oracle Financial Services Software Ltd.	Oraclefin	532466
Patni Computer Systems Ltd.	Patni	532517
Tata Consultancy Services Ltd.	Tcs	532540
Wipro Ltd.	Wipro	507685

The stocks included in the BSE-METAL index are:

Company Name	Scrip Id	Scrip Code
Bhushan Steel Ltd.	Bhussteel	500055
Hindalco Industries Ltd.	Hindalco	500440
Hindustan Zinc Ltd.	Hindzinc	500188
Jindal Saw Ltd.	Jindal	500378
National Aluminium Co.Ltd.	Nalco	532234
Nmdc Ltd.	Nmdcltd	526371
Steel Authority Of India Ltd.	Sail	500113
Sesa Goa Ltd.	Sesagoa	500295
Sterlite Industries (India) Ltd.	Sterliteind	500900
Tata Steel Ltd.	Tatastl	500470
Welspun Corp Ltd.	Welcorp	532144

Due to economic recession, the data taken during the periods 2007-2009 were giving very low returns and hence results were not displayed in this paper. Sectors BSE-POWER and BSE-BANKEX were not considered due to lack of data for the selected period.